mathematics

Review

Roles and Research Trends of Artificial Intelligence in Mathematics Education: A Bibliometric Mapping Analysis and Systematic Review

Gwo-Jen Hwang * and **Yun-Fang Tu**

Graduate Institute of Digital Learning and Education, National Taiwan University of Science and Technology, Taipei City 106335, Taiwan; sandy0692@gmail.com
* Correspondence: gjhwang@mail.ntust.edu.tw

Abstract: Learning mathematics has been considered as a great challenge for many students. The advancement of computer technologies, in particular, artificial intelligence (AI), provides an opportunity to cope with this problem by diagnosing individual students' learning problems and providing personalized supports to maximize their learning performances in mathematics courses. However, there is a lack of reviews from diverse perspectives to help researchers, especially novices, gain a whole picture of the research of AI in mathematics education. To this end, this research aims to conduct a bibliometric mapping analysis and systematic review to explore the role and research trends of AI in mathematics education by searching for the relevant articles published in the quality journals indexed by the Social Sciences Citation Index (SSCI) from the Web of Science (WOS) database. Moreover, by referring to the technology-based learning model, several dimensions of AI in mathematics education research, such as the application domains, participants, research methods, adopted technologies, research issues and the roles of AI as well as the citation and co-citation relationships, are taken into account. Accordingly, the advancements of AI in mathematics education research are reported, and potential research topics for future research are recommended.

Keywords: artificial intelligence; mathematics education; bibliometric mapping analysis; systematic review

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1. Introduction

Mathematics refers to the learning content which employs symbolic language to represent such concepts as number, quantity, space and structure [1]. Mathematics education has been identified as a complex and challenging task aiming to foster learners' problem-solving competence [2]. Several previous studies have reported that students generally feel that it is difficult to complete mathematics tasks, in particular, those which need to be resolved with multiple steps [3,4]. Therefore, researchers have made attempts to develop various learning strategies and tools to enhance students' mathematics learning outcomes [1]. They have also pointed out the importance of identifying the factors affecting students' mathematics learning performance, such as insufficient prior knowledge and lack of personalized supports for individual students [5,6].

In the meantime, the advancement of artificial intelligence (AI) has provided a means to deal with these problems [7]. AI refers to the field of computer science research aimed at developing computer systems that are capable of performing tasks requiring human intelligence, such as visual and voice recognition, inferencing and decision making [8]. Several previous applications have revealed the potential of applying AI in education, especially for helping students face complex or challenging tasks [9,10]. For example, Chen and Liu [11] developed a personalized computer-assisted mathematics problem-solving system and found it effective for improving students' learning performance and attitude.

Researchers have identified several roles of AI in education, such as an intelligent tutor, tutee, learning tool and partner as well as advisor for making educational policies [12]. As for the role of intelligent tutor, the use of AI technologies to simulate teachers' intelligence for providing personalized guidance, feedback or supports to individual students during the learning process has been demonstrated by several researchers. For example, Hwang et al. [13] developed an adaptive learning system for mathematics courses by taking into account individual students' cognitive and affective performances.

In order to examine the roles and research trends of AI in mathematics education (AIME), the present study conducted bibliometric mapping analysis and a systematic review to analyze the studies published in the WOS database following the technology-based learning model proposed by Lin and Hwang [14] to answer the following research questions:

1. What and who are the major journals publishing the AIME studies? What are the most cited papers of AIME research? Who are the most productive and cited authors of AIME research?
2. What are the most used keywords of AIME research? What are the relationships between the keywords?
3. What are the application domains of AIME research?
4. What are the sample groups selected for AIME research?
5. What are the research methods adopted in AIME research?
6. What are the roles of AI in mathematics education?
7. What are the adopted AI algorithms in AIME research?
8. What are the research issues investigated in AIME research?

2. Literature Review

The advancement of various information, communication and computing technologies has provided new opportunities for improving teaching and learning; in particular, the rapid advance of AI enables computer systems to act more like a tutor than conventional tutoring systems [7]. AI technologies can be used to analyze students' learning process, including interaction content, learning behaviors, test results and learning perceptions, to provide instant support or feedback to individual students as well as suggestions to teachers for improving teaching content and strategies [15]. Scholars [11,12,15] have indicated that facilitating personalized learning is among the key objectives of Artificial Intelligence in Education (AIED). Zawacki-Richter et al. [16] reviewed the AIED in higher education studies published from 2007 to 2018 and concluded that AI has been applied to various application domains, in particular, computer science, science, technology, engineering and mathematics.

Researchers have also indicated that, in the 21st century, in addition to delivering knowledge, it is important to foster students' higher order thinking, such as questioning, critical-thinking, problem-solving and creative-thinking abilities; mathematics is the foundation of these abilities [17]. Several previous studies have emphasized that in mathematics education, it is important to support students to learn to think critically, communicate with others, solve problems and construct knowledge, while also delivering mathematics concepts and methods to them [18,19]. Several scholars [7,16,20] have further pointed out that the use of AI technologies to analyze students' learning status or behaviors makes it possible to develop intelligent tutors, which are able to provide effective interventions to individual students to improve their learning performances and motivation. For example, one of the studies [21] employed the genetic algorithm to implement a personalized e-learning system to provide personalized curriculum sequencing recommendations to individual learners to promote their learning performances.

Furthermore, the incorporation of AI technologies into educational settings enables computer-based learning systems to play roles of intelligent tutors, tools or tutees as well as policy-making facilitators [12,20]. For example, some previous studies employed AI technologies to simulate the behaviors of teachers in diagnosing students' learning problems and providing personalized learning content and paths as well as suggestions or guidance to individual students in mathematics courses [22–24]. A recent review study regarding technology-enhanced adaptive/personalized learning [25] reported that the advancement and popularity of AI has gradually accomplished an important objective of technology-enhanced learning, that is, providing personalized or adaptive learning environments to improve students' learning achievements. For example, some studies have reported that the provision of context personalization in intelligent tutoring systems (ITS) can promote learners' situational interest and performance in math tasks [26,27]. Another example is the use of AI technologies (e.g., unsupervised machine learning method) in developing student models for predicting individual students' learning engagement or status in mathematics courses [28].

From the literature, it was found that AI is becoming increasingly influential in mathematics education. Scholars have indicated that, via analyzing the publications in a specific domain, valuable information regarding the trends or potential research issues can be provided to researchers in the field [12,16,25]. In the past three decades, researchers have mainly paid attention to the trends and issues of AIED [7,12], AI in e-learning [29], AI in higher education [16], AI in medical education [30] and AI in engineering applications [31]. Scholars have pointed out that mathematics education is very important in the 21st century since it is highly related to the development of students' problem-solving competence and cross-curricular experiences [32–34]. Gallagher et al. [32] conducted a literature review on the issue of adaptive teaching in mathematics from 1975 to 2014, and pointed out that technology could assist students in the process of learning mathematics knowledge and skills, and could cultivate their creativity. However, there has not yet been a review for AIME.

To cope with this problem, this research aims to use bibliometric mapping analysis to analyze AIME research, including the most frequently adopted keywords, the most contributing journals, papers and authors. We further conduct a systematic review and discuss the dimensions of application domains, sample groups, research methods, roles of AI, AI algorithms and research issues based on the technology-based learning model [14,35,36].

3. Method

3.1. The Article Selection Process

On 31 December 2020, we searched the publications in the "education/educational research" category from the WOS database using two substrings of keywords: "AI" ("artificial intelligence" or "machine intelligence" or "intelligent support" or "intelligent virtual reality" or "chat bot*" or "machine learning" or "automated tutor*" or "personal tutor*" or "intelligent agent*" or "expert system*" or "neural network*" or "natural language processing" or "chatbot*" or "intelligent system" or "intelligent tutor*") [16] and "mathematics education" ("mathematics" or "math" or "statistics" or "calculus" or "algebra") [19]. A total of 136 articles were obtained. By excluding non-article types, 129 articles were retained. Following that, a manual review was conducted to examine the content of each article (including paper title and abstract) to eliminate repeated, non-English, literature review and irrelevant publications to ensure that the selected articles involved the use of AI in practical mathematics learning activities. Finally, 43 articles were retained for content and bibliometric mapping analysis (see Figure 1).

3.2. Data Coding and Analysis

The first three research questions were answered by employing the bibliometric mapping analysis. The VOSviewer software was adopted to analyze the citation, co-citation and the most used author keywords in the articles.

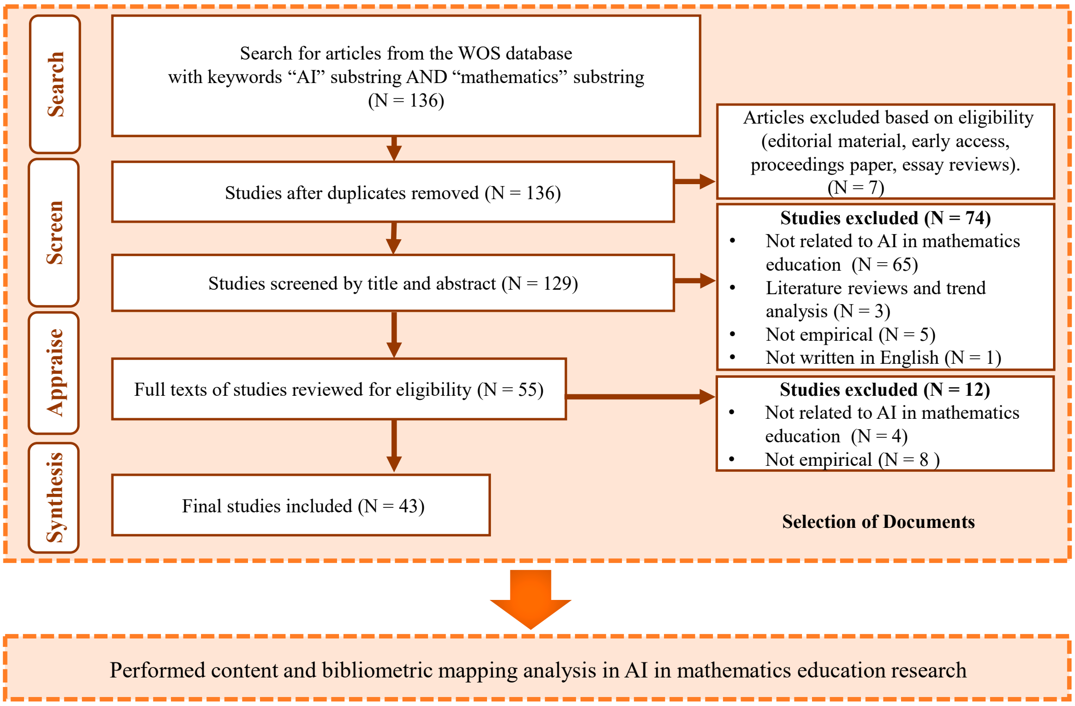


Figure 1. Article selection process for bibliometric mapping analysis and systematic review.

To answer the other research questions, a systematic review was conducted by referring to the theoretical model and coding scheme of Hsu et al. [35], Lin and Hwang [14] and Tu and Hwang [36]. As shown in Figure 2, several dimensions (i.e., application domains, sample groups, research methods, roles of AI, AI algorithms and research issues); journals; most-cited papers and authors; and most used keywords were taken into account. Accordingly, the coding scheme for each dimension is listed as follows:

1. Application domains: by referring to Yang et al. [19], the application domains in mathematics education were categorized into general/ mathematics foundations, discrete mathematics /algebra, analysis, geometry/ topology and applied mathematics, others, non- specified and mixed.
2. Research sample groups: by referring to Hsu et al. [35], the research sample groups in the literature were categorized into elementary school students, junior high school students, higher education students, teachers, mixed groups and non-specified.
3. Research methods: by referring to the coding scheme of Hsu et al. [35], the research methods were divided into quantitative, qualitative and mixed methods.
4. Roles of AI: as suggested by Zawacki-Richter et al. [16], the roles of AI in education include profiling and prediction, ITS, assessment and evaluation and adaptive systems and personalization.
5. Adopted AI algorithms: by referring to the study of Hwang et al. [12], AI algorithms were categorized into evolutionary algorithms, Bayesian inferencing and networks, search and optimization, fuzzy set theory, deep learning, case-based reasoning, traditional machine learning approaches and knowledge elicitation methods via interviewing domain experts and mixed. Traditional machine learning approaches include statistical learning; data mining; or symbolic learning approaches, such as Item Response Theory (IRT), linear regression, polynomial regression, classification, clustering, Iterative Dichotomiser 3 (ID3), version space, support vector machines and neural networks.
6. Research issues: by referring to Tu and Hwang [36], research issues were classified into cognitive, affect, skills, learning behaviors, correlation, relevance, system design or evaluating AI system/tool performance, meta-cognition and learning styles.

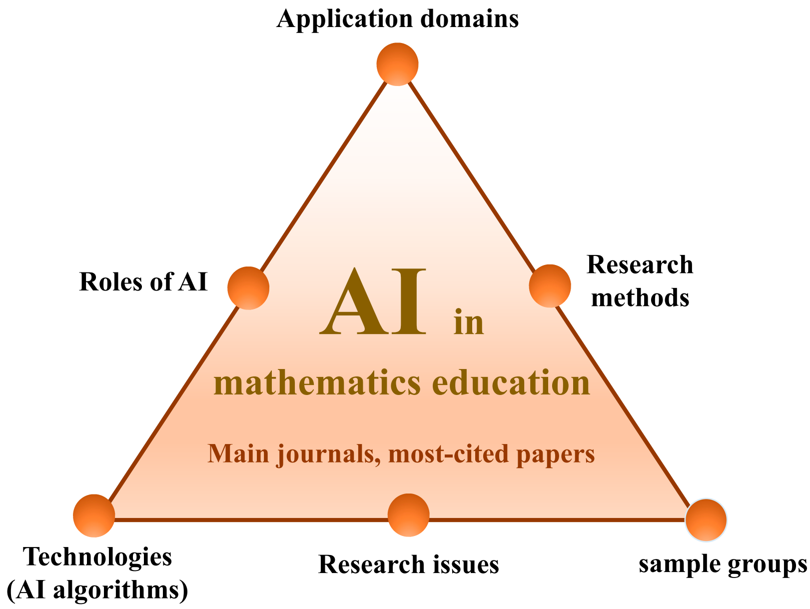


Figure 2. Model for reviewing artificial intelligence (AI) in mathematics education (AIME) research.

The coding was performed by two experienced researchers who read and categorized the AIME articles based on the coding scheme. The kappa value of the two researchers' coding results was 0.85, showing a high consistency [37].

3.3. Data Coding and Analysis

Figure 3 illustrates the number of AIME publications in each year from 1996 to 2020. Based on the suggestions of several previous studies to take into account the fluctuation of technology [16,36,38], the AIME studies are categorized into three time periods, that is, 1996–2010, 2011–2015 and 2016–2020. Accordingly, there are six publications from 1996 to 2010, 12 from 2011 to 2015 and 25 published papers from 2016 to 2020, as shown in Figure 3. It was found that the number of publications in the latter two periods was nearly twice that of the previous time period, showing the rapid growth of AIME research in the past decade. This finding could be related to the advancement of computer and AI technologies in the past 10 years.

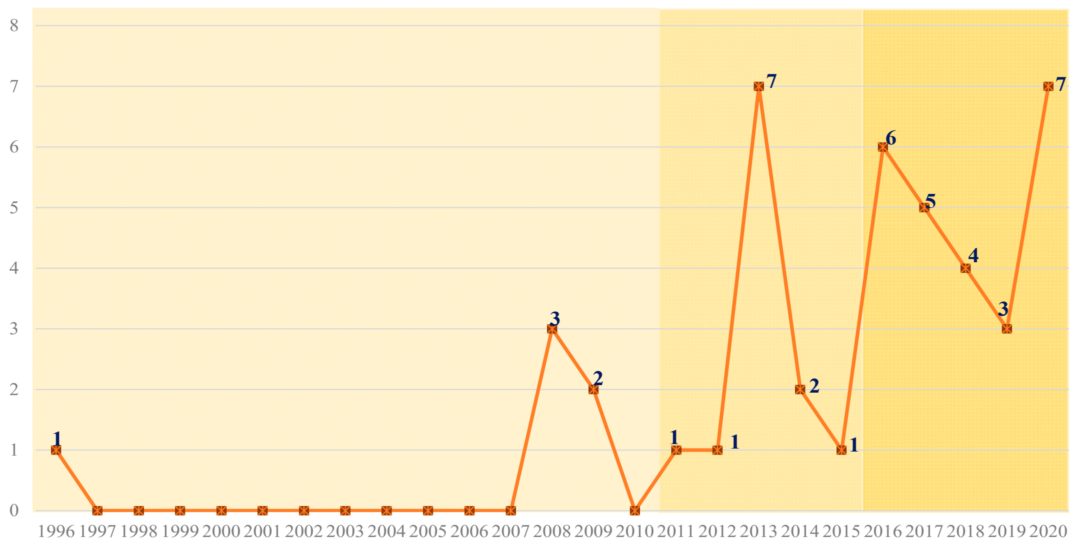
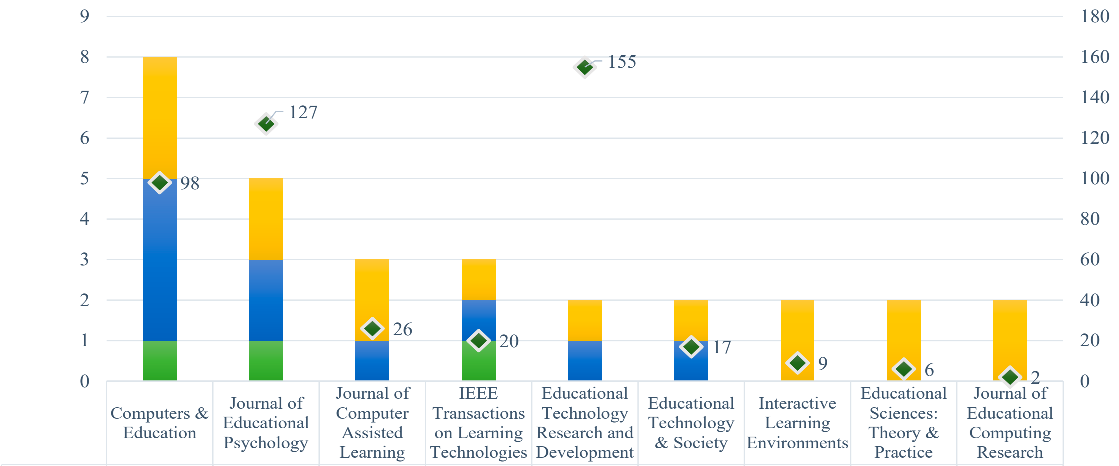


Figure 3. Published literature on AI in mathematics education from 1996 to 2020.

4. Results

4.1. Main Journals, Most Cited Papers and Most Productive and Cited Authors

Figure 4 shows nine journals with the largest number of articles in AIME research between 1996 and 2020. They were *Computers & Education* (publications = 8), *Journal of Educational Psychology* (publications = 5), *Journal of Computer Assisted Learning* (publications = 3), *IEEE Transactions on Learning Technologies* (publications = 3), *Educational Technology Research and Development* (publications = 2), *Educational Technology & Society* (publications = 2), *Interactive Learning Environments* (publications = 2), *Educational Sciences: Theory & Practice* (publications = 2) and *Journal of Educational Computing Research* (publications = 2). It also shows that the most cited journals are *Educational Technology & Society* (citations = 155), *Journal of Educational Psychology* (citations = 127) and *Computers & Education* (citations = 98).



	Computers & Education	Journal of Educational Psychology	Journal of Computer Assisted Learning	IEEE Transactions on Learning Technologies	Educational Technology Research and Development	Educational Technology & Society	Interactive Learning Environments	Educational Sciences: Theory & Practice	Journal of Educational Computing Research
2016–2020	3	2	2	1	1	1	2	2	2
2011–2015	4	2	1	1	1	1	0	0	0
1996–2010	1	1	0	1	0	0	0	0	0
Citations	98	127	26	20	155	17	9	6	2

Figure 4. Top 9 journals by total number of publications from 1996 to 2020 (note: only journals with 2 or more publications are included).

In addition, co-citation analysis and cited sources were selected. The minimum number of citations from sources was adjusted to 10, and the number of sources to be selected was automatically displayed as 30. Figure 5 shows that the top three most cited journals are *Journal of Educational Psychology* (79 co-citations), *Computers & Education* (72 co-citations) and *Learning and Instruction* (34 co-citations).

Table 1 shows the top three most-cited papers, which were published by *Educational Technology Research and Development, Journal of Educational Psychology and Innovations in Education and Teaching International*, respectively. This more or less indicates that the journals have taken the studies of AI and mathematics education as important research foci.

Table 1. Top 3 most-cited papers.

Rank	Title	Journal	Authors, Year	Total # of Citations
1	Are badges useful in education?: it depends upon the type of badge and expertise of learner	*Educational Technology Research and Development*	Abramovich, Schunn and Higashi, 2013	154
2	Using Adaptive Learning Technologies to Personalize Instruction to Student Interests: The Impact of Relevant Contexts on Performance and Learning Outcomes	*Journal of Educational Psychology*	Walkington, 2013	65
3	Diagnosing student learning problems based on historical assessment records	*Innovations in Education and Teaching International*	Hwang, Tseng and Hwang, 2008	39

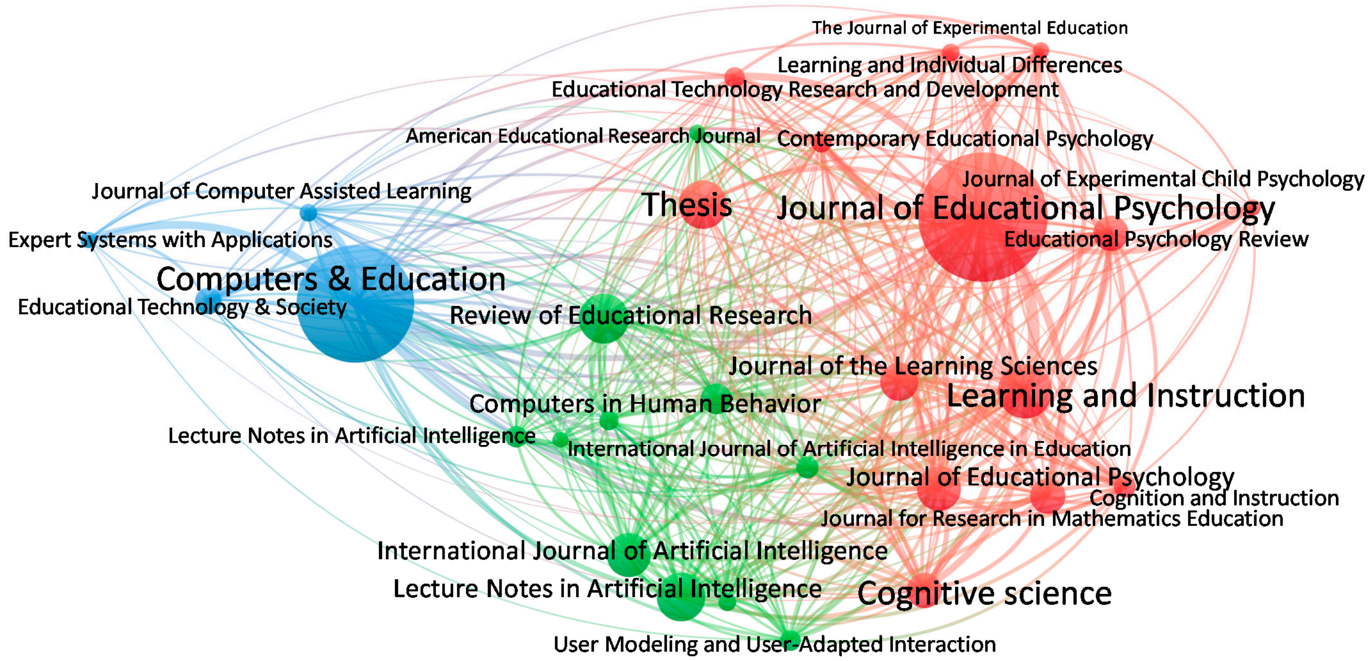


Figure 5. The most cited journals (co-citation analysis).

The first study by Abramovich et al. [39] investigated the effects of using different badges on students' learning motivation with ITS in mathematics courses, showing the potential of incorporating gamification or award (i.e., badges) strategies into AI-based systems. The second study by Walkington [40] suggested that the use of adaptive technologies to provide personalized instructions had good potential to promote students' learning interest, and hence improve their learning performances. The third by Hwang et al. [41] reported the effectiveness of using AI technologies to diagnose students' learning problems and provide personalized learning suggestions to individual students in a mathematics course.

Table 2 shows the authors who have published two or more AIME studies. The top three authors with the highest number of citations were Xiangen Hu (citations = 49, publications = 3), Gwo-Jen Hwang (citations = 48, publications = 2) and Scotty D. Craig (citations = 40, publications = 2).

Table 2. Top authors ranked by number of publications.

Author	Countries/Areas	Publications	Total # of Citations (Citations Per Paper)
Xiangen Hu	USA	3	49 (16.33)
Candace Walkington	USA	3	33 (11)
Gwo-Jen Hwang	Taiwan	2	48 (24)
Scotty D. Craig	USA	2	40 (20)
Vincent Aleven	USA	2	22 (11)

Figure 6 shows the co-citation analysis results by setting the minimum number of citations as 10. It was found that the publications by Koedinger (30 citations), Graesser (23 citations) and Walkington (22 citations) have been co-cited the most in AIME research.

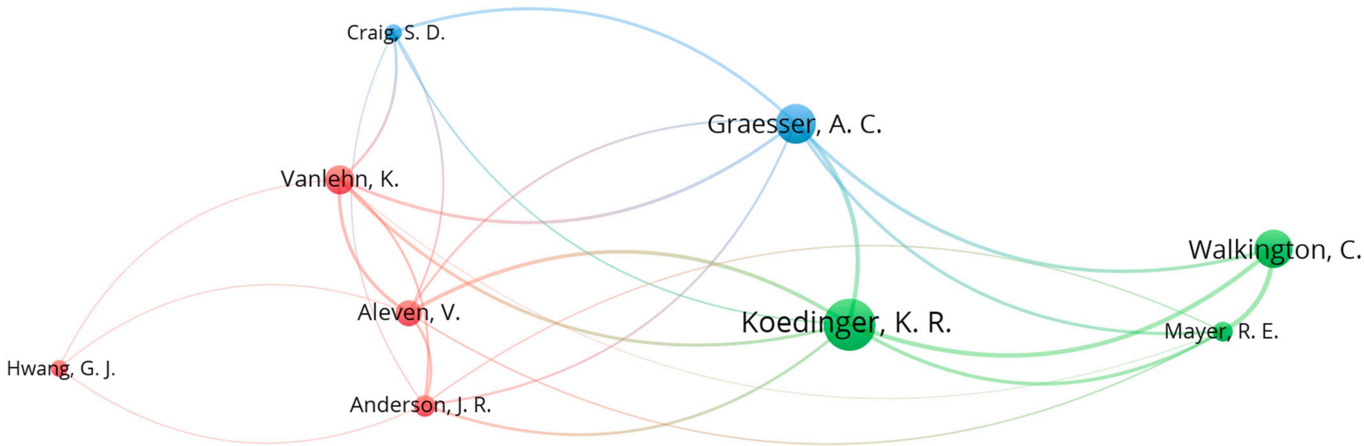


Figure 6. The most cited authors (co-citation analysis).

4.2. Most Used Keywords

A total of 163 author keywords are included in the 43 AIME articles. Figure 7 shows the cluster analysis results generated by VOSviewer, including the dynamic change and network map. The most frequently used keywords are "intelligent tutoring system" (f = 14), "mathematics education" (f = 5), "personalization" (f = 4), "algebra" (f = 3), "feedback" (f = 3), "human-computer interface" (f = 3), "intelligent tutors" (f = 3), "interactive learning environments" (f = 3) and "learning diagnosis" (f = 3).

In addition to mathematics education, Figure 7A shows that the popular author keywords in recent studies are personalization, feedback, human-computer interface and so on. This implies that the main focus of AIME research is to provide personalized learning support or guidance in learner-centered contexts.

Figure 7B shows three main clusters of AIME research, that is, "AI-based learning systems", "personalized/adaptive learning" and "learning strategies/models", as displayed in red, green and blue. The studies in Cluster 1 (i.e., AI-based learning systems) focus on the development and applications of AI technologies and ITS to improve students' learning performances. Among the three clusters, Cluster 1 includes the earliest AIME research. For example, Bennett and Sebrechts [42] evaluated the accuracy of the automatic qualitative judgments generated by an expert system in diagnosing students' problems when learning algebra. Cluster 2 focuses on providing personalized/adaptive learning environments using AI technologies. For example, Wang et al. [43] demonstrated an adaptive learning system which can provide personalized learning content based on individual students' learning statuses to increase their learning gains. Cluster 3 focuses on incorporating various learning strategies into AI-supported learning environments to improve students' learning outcomes in mathematics courses. For example, Jiménez-Hernández et al. [44] proposed a gamification approach in an AIME learning environment to enhance the students' learning motivation and achievement.

In addition to the three clusters, it is worth noting another research focus, that is, educational data mining (EDM), such as clustering, classification, Bayesian modeling, relationship mining and discovery with models. EDM can be used to help policy makers or administrators in educational institutes determine important policies for mathematics education [45]. It can also be used to analyze students' class attendance, learning status and homework submission materials to predict the potential drop-out risk of individual students [7] and the factors affecting students' mathematics literacy [46].

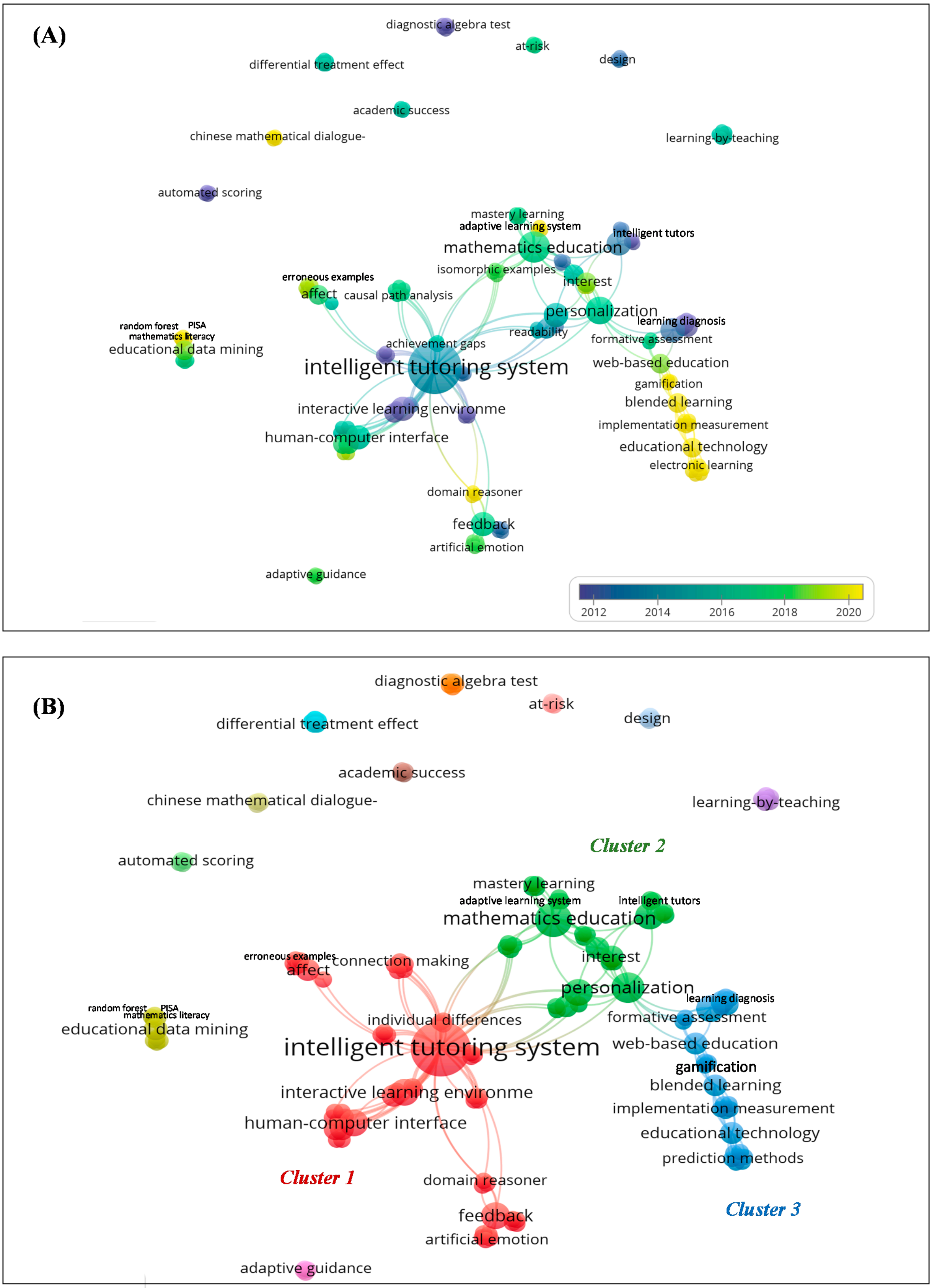
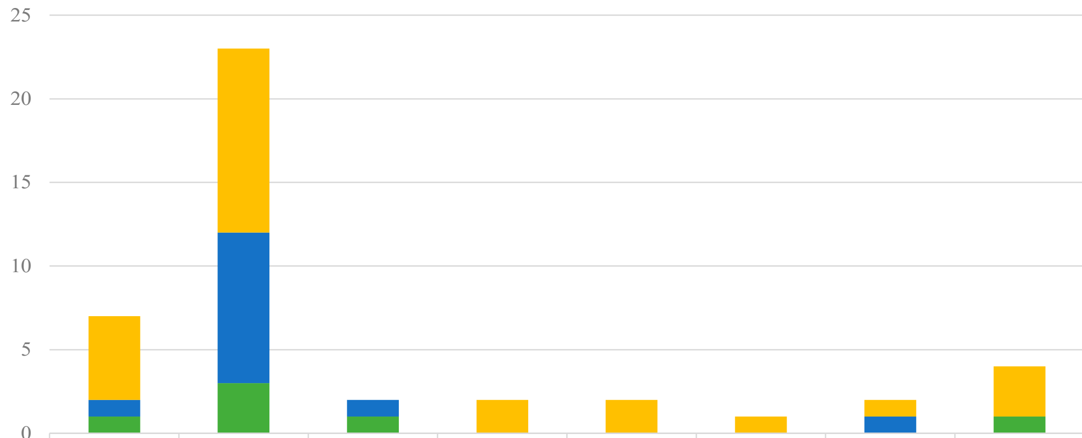


Figure 7. Analysis of keyword co-occurrence between 1997 and 2019. (**A**) The distribution of the AIME research using the keyword by years. (**B**) The most used keyword in the AIME research.

4.3. Application Domains

Figure 8 shows the number of application domains in individual time periods. It was found that the most frequent applications of AIME are discrete mathematics/algebra (53.49%), followed by general/foundations (16.28%) and mixed (9.30%). Moreover, the number of individual applications generally increased, and the application domains became more diverse from the first to the third time periods.

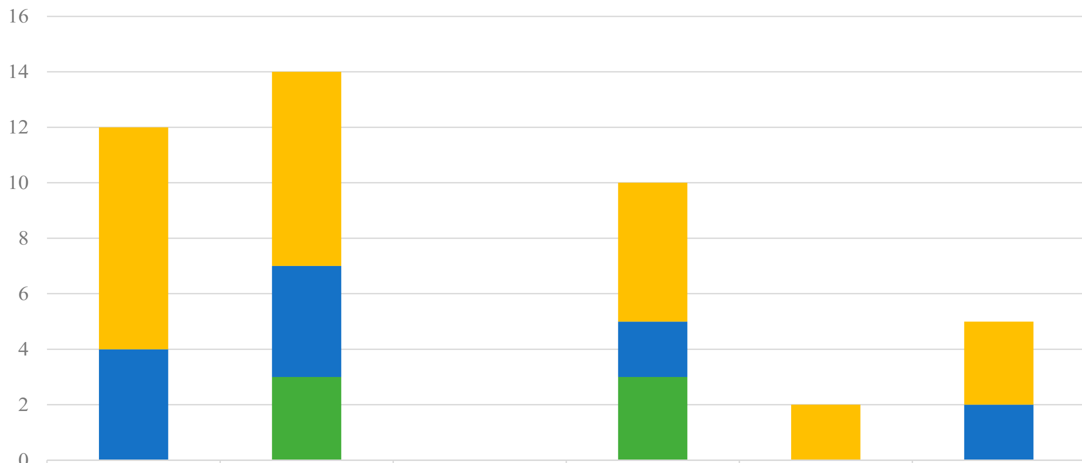


	General/ foundations	Discrete mathematics /algebra	Analysis	Geometry and topology	Applied mathematics	Mathematics literacy	Non-specified	Mixed
2016–2020	5	11	0	2	2	1	1	3
2011–2015	1	9	1	0	0	0	1	0
1996–2010	1	3	1	0	0	0	0	1

Figure 8. Application domains of AIME research in each period.

4.4. Sample Groups

Figure 9 shows the sample groups adopted by the AIME studies. It was found that junior high school students were the most frequently adopted samples (32.56%), followed by elementary school students (27.91%) and higher education students (23.26%). It was also found that no senior high school students were adopted in those AIME studies; moreover, two studies adopted teachers as sample groups in the third time period.



	Elementary school students	Junior high school students	Senior high school students	Higher education students	Teachers	Mixed
2016–2020	8	7	0	5	2	3
2011–2015	4	4	0	2	0	2
1996–2010	0	3	0	3	0	0

Figure 9. Research sample groups of AIME research in each period.

4.5. Research Methods

Figure 10 shows the research methods adopted by the AIME studies. Quantitative methods were adopted the most (79.07%), followed by mixed methods (18.60%) and qualitative methods (2.33%) in each time period.

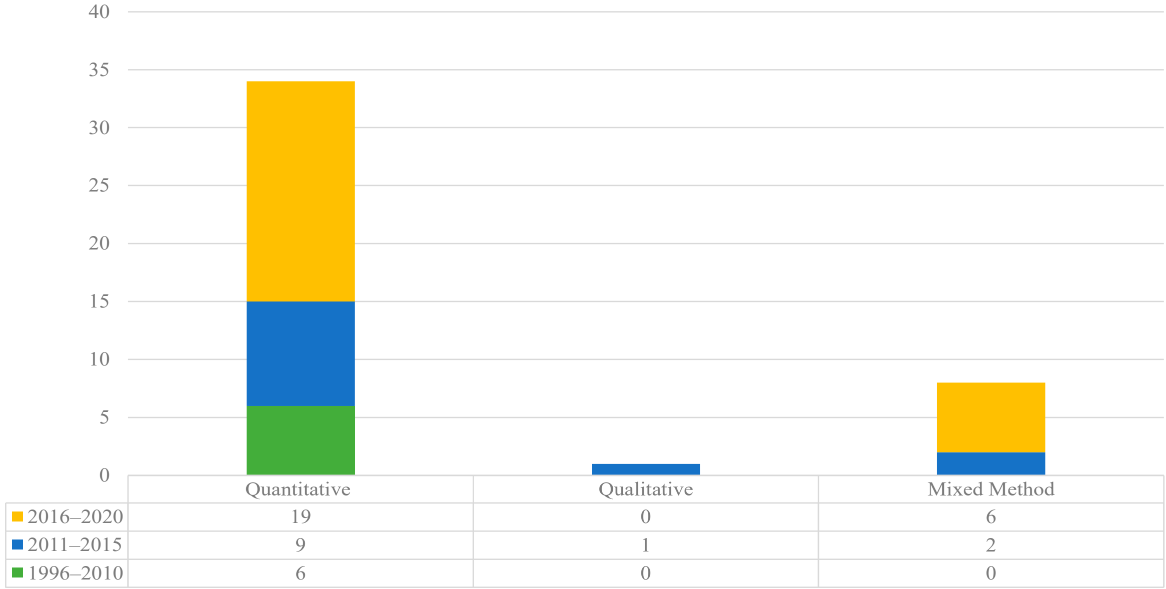


Figure 10. Research methods of AIME research in each period.

4.6. Roles of AI

Figure 11 shows the roles of AI in the AIME research. The most frequent role played by AI is "intelligent tutoring systems" (45.24%), followed by "profiling and prediction" (28.57%) and "adaptive systems and personalization" (21.43%). Generally speaking, for each role of AI, the number of studies increased from the first to the third time periods, except for the role of "assessment and evaluation."

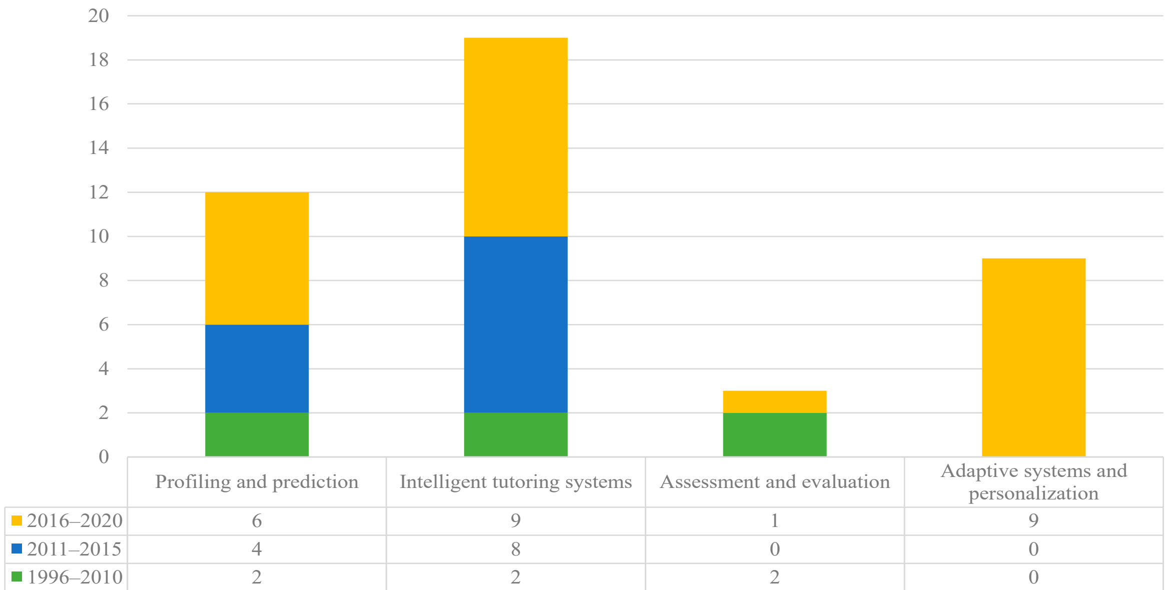


Figure 11. Roles of AIME research in each period.

Figure 12 shows the relationships among sample groups, roles of AI and the educational supports provided by AI. For example, it was found that the main educational supports provided by "intelligent tutoring systems (ITS)" were "student models and academic achievement" (25.58%), followed by "diagnosing strengths and automated feedback" (18.60%) and "teacher's perspective (i.e., reducing teachers' loadings" (18.60%); moreover, these ITS studies focused on diagnosing the mathematics learning problems of elementary school and junior high school students, and providing feedback to them so as to reduce the teachers' loadings (e.g., [47–49]). On the other hand, it was found that the issues related to curating learning materials for individual students and facilitating collaboration were seldom investigated in ITS studies.

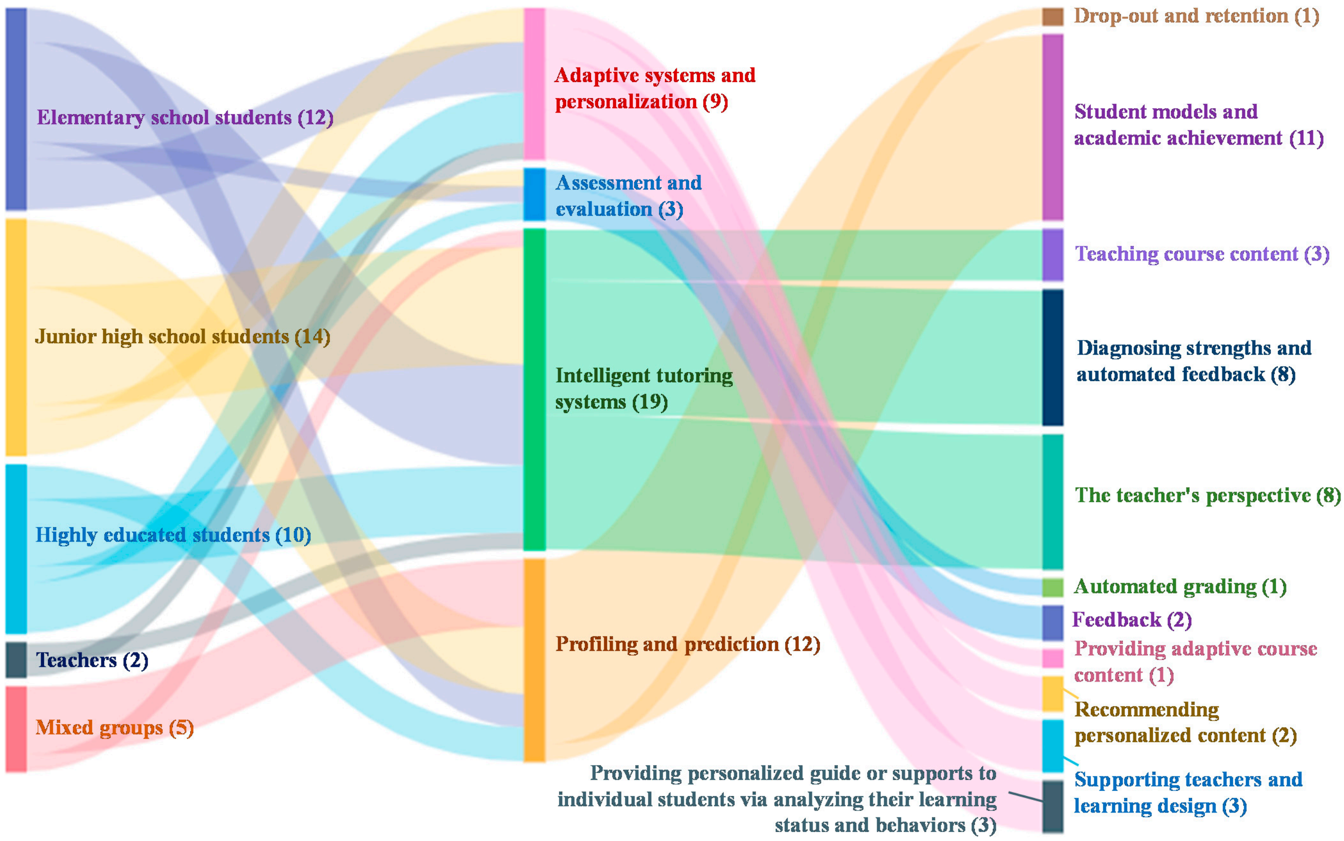


Figure 12. Relationships among sample groups, roles of AI and the educational supports provided by AI.

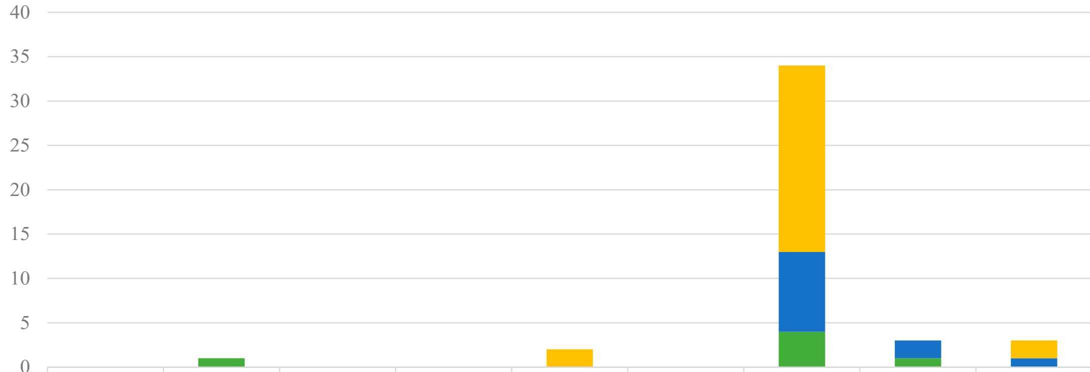
In the studies in which the role of AI was "profiling and prediction", the sample groups were mainly junior high school students and mixed groups, and the supports from AI were mainly finding student models for improving students' academic achievement [50–52]).

As for the role "adaptive systems and personalization", the relevant studies were published in 2016–2020 to provide personalized guidance or supports to almost all levels of students (i.e., elementary school, junior high school and higher education) via analyzing their learning status and behaviors (e.g., [43,53–55]). It was also found that these "adaptive systems and personalization" studies seldom refer to the diagnosis of students' learning problems for providing proactive personal guidance.

The role of "assessment and evaluation" was applied to the studies for higher education in the earlier stage and was used in the studies for elementary school students in recent years for automated grading [42] and feedback [28]. In the studies related to this role of AI, the issues related to the evaluation of student understanding, engagement and academic integrity as well as the evaluation of teaching were rarely included.

4.7. Adopted AI Algorithms

Figure 13 shows the adopted AI algorithms. It was found that most studies adopted the traditional machine learning approach (79.07%), followed by knowledge elicitation methods via interviewing domain experts (6.98%), mixed (6.98%) and deep learning (4.65%), while evolutionary algorithms, search and optimization, fuzzy set theory and case-based reasoning were not adopted in the studies.



	Evolutionary algorithms	Bayesian inferencing and networks	Search and optimization	Fuzzy set theory	Deep learning	Case-based reasoning	Traditional machine learning approach	Knowledge elicitation methods via interviewing domain experts	Mixed
2016–2020	0	0	0	0	2	0	21	0	2
2011–2015	0	0	0	0	0	0	9	2	1
1996–2010	0	1	0	0	0	0	4	1	0

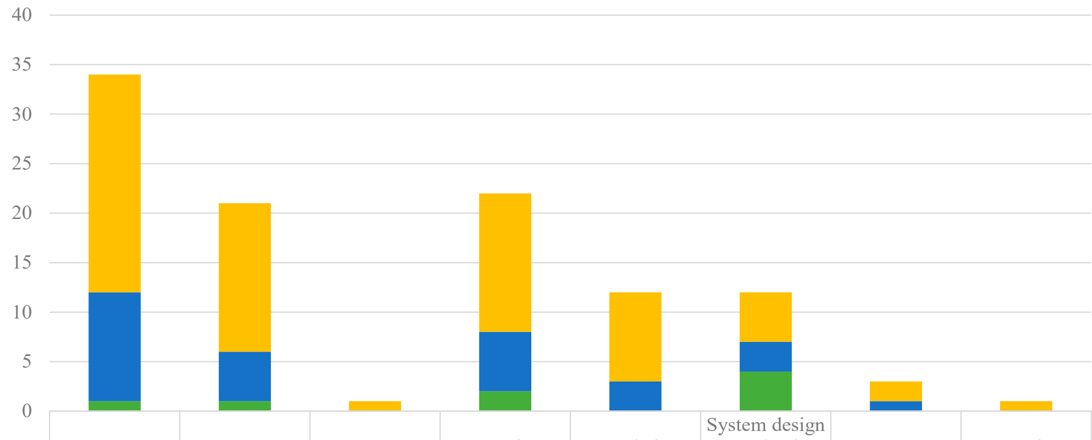
Figure 13. Adopted technologies of AIME research in each period.

Those studies which adopted traditional machine learning approaches were related to the development and application of ITS (e.g., [39,47,48]). The use of knowledge elicitation methods via interviewing domain experts was also related to the development of intelligent learning systems for assessing students' learning status and providing feedback to them (e.g., [40,41]). Deep learning or other algorithms were generally used for building models for predicting students' learning behaviors or performances (e.g., [45,51]).

4.8. Research Issues

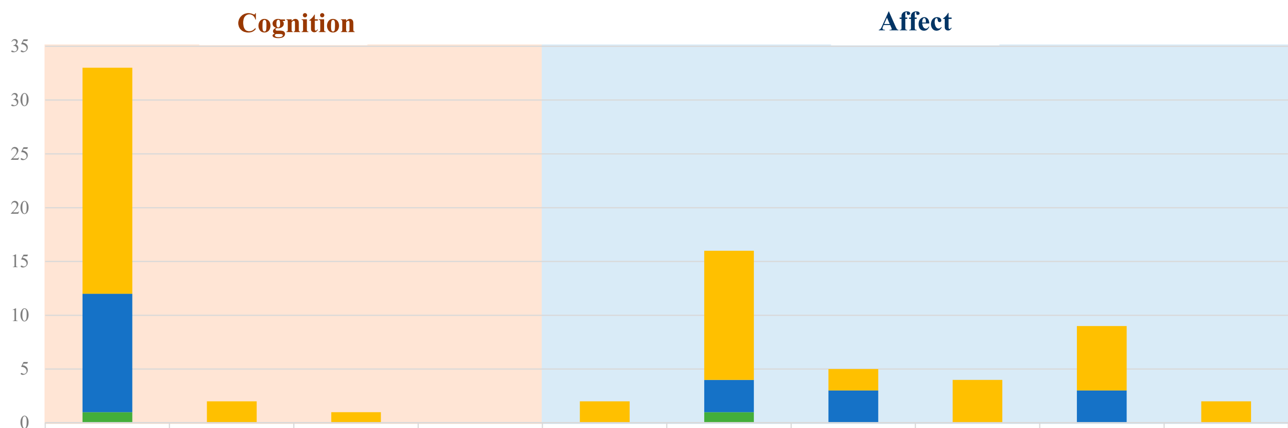
Figure 14 shows the number of different research issues investigated in the AIME research. It should be noted that generally two or more research issues were investigated in each study. The analysis results show that the issues related to cognition were investigated the most (34 articles), followed by learning behaviors (22 articles) and affect (21 articles). It was also found that the research issues became more diverse from the first time period to the third. For example, in the third time period, the issues related to skill (e.g., [56]), meta-cognition (e.g., [57]) and learning styles (e.g., [54]) were included.

Figure 15 further shows the cognition and affect issues investigated in the AIME research. In terms of cognition, most studies measured students' learning performances (33 articles), while few considered students' higher order skills (2 articles) or collaboration or communication (1 article). Moreover, none of the studies investigated students' cognitive load. In terms of affect, most studies investigated the students' attitude or motivation (16 articles), followed by learning perceptions (9 articles), self-efficacy (5 articles) and satisfaction (4 articles).



	Cognition	Affect	Skills	Learning behaviors	Correlation, relevance	System design or evaluating AI system/tool performance	Meta-cognition	Learning styles
2016–2020	22	15	1	14	9	5	2	1
2011–2015	11	5	0	6	3	3	1	0
1996–2010	1	1	0	2	0	4	0	0

Figure 14. Research issues of AIME research in each period.



	Learning performance	Higher order skills	Collaboration or communication	Cognitive load	Technology acceptance model or intention of use	Attitude, motivation	Self-efficacy or confidence	Satisfaction or interest	Opinion of learner or learning perception	Anxiety
2016–2020	21	2	1	0	2	12	2	4	6	2
2011–2015	11	0	0	0	0	3	3	0	3	0
1996–2010	1	0	0	0	0	1	0	0	0	0

Figure 15. Research issues in the cognition and affect aspects.

In the cognition dimension, early studies mainly focused on measuring students' learning performance. In the third time period, several studies started to investigate students' higher order thinking as well their collaboration and communication competences. For example, one study aimed to use ITS to facilitate students' construction of mathematical ideas to help them understand the reasoning behind mathematics to improve their mathematics problem solving [58]. On the other hand, it was found that cognitive load was not discussed in those AIME studies.

In the affect dimension, learning attitudes and motivations were the most frequently investigated issues in the early studies. Moreover, in recent years, the investigated issues have become more diverse; for example, Güre et al. [46] used educational data mining to analyze the data of the Programme for International Student Assessment (PISA) 2015 in Turkey and found that more family supports could reduce students' mathematics anxiety.

5. Discussion

This study analyzed 43 articles of AIME published between 1996 and 2020 in the WOS database. Several studies have demonstrated that the use of AI technology has great potential for promoting students' learning performances and higher order thinking [59]. In addition, using AI technology to diagnose students' learning problems can not only provide instant feedback to individual students but can also provide information to help teachers improve the learning design [7,60,61]. From the analysis results, the following findings and implications were derived:

- The greatest amount of AIME research was published in *Computers & Education*, followed by the *Journal of Educational Psychology* and the *Journal of Computer Assisted Learning*. In addition, the top three most cited journals (co-citation analysis) are the *Journal of Educational Psychology, Computers & Education* and *Learning and Instruction*. That is, more education and educational technology researchers have engaged in AIME research than mathematics education researchers. This implies the need to encourage mathematics education researchers to consider using AI technology in their studies.
- From the results of using cluster analysis on author keywords, three clusters of AIME studies were found; that is, "AI-based learning systems", "personalized/adaptive learning" and "learning strategies/models." Moreover, a new and small cluster, EDM in mathematics education, was formed in recent years. This could be a good reference for those intending to conduct AIME research in the future.
- The most frequently adopted application for AIME studies was discrete mathematics/algebra, followed by general/foundations. On the other hand, geometry and topology, applied mathematics, mathematics literacy and across-disciplines (e.g., STEM) were seldom included in those AIME studies. This implies that AIME applications remain in the beginning stage; that is, researchers mainly focused on using AI technologies to solve fundamental problems in mathematics courses.
- The most frequently adopted sample group for AIME studies was junior high school students, followed by elementary school students and higher education students. On the other hand, teachers and senior high school students were seldom adopted by AIME research. This could be due to the fact that learning mathematics in junior high school is more challenging than in elementary school. Therefore, junior high school students need more assistance to face the challenge. Moreover, choosing elementary school students and higher education students could be due to convenience. Elementary school teachers generally tend to accept new learning approaches, since they need not worry about students' entrance examinations, in particular, in Asian countries. Choosing higher education as the sample groups is also a convenient selection, since most of the authors were researchers in universities. Similarly, most studies focused on students' learning performance, since it is the main objective for all levels of mathematics education.
- Quantitative methods were the most frequently adopted approaches, followed by mixed methods. This is reasonable, since most studies aimed to evaluate students' learning performance via analyzing their test scores as well as learning attitudes or attitudes via questionnaires.
- The most frequent role played by AI in mathematics education was "intelligent tutoring systems", followed by "profiling and prediction" and "adaptive systems and personalization." This is consistent with the finding regarding the research issue, that is, evaluating students' learning performance is the main focus of AIME studies. The main purpose of developing ITS is to evaluate students' learning problems and to provide instant supports to them, which aims to improve their learning performances. Although adaptive learning systems and personalization have the same aim, developing such adaptive learning systems is more challenging, and hence the number of such studies is relatively small.

- Most studies adopted the traditional machine learning approach, or knowledge elicitation methods via interviewing domain experts, while modern AI approaches, such as deep learning, were seldom adopted. This could be due to the fact that those AIME studies mainly focus on the development of ITS for evaluating individual students' learning statuses to provide assistance to them. This objective is highly related to features of traditional machine learning approaches (e.g., statistical learning, data mining and decision trees) and knowledge elicitation methods via interviewing domain experts; that is, domain knowledge is explicitly represented and used for decision making or prediction [7,12].
- Most AIME studies investigated students' learning achievements (cognition dimension), and learning motivation and attitude (affect dimension). This is because the objective of mathematics education is to foster students' cognition competences. Moreover, since mathematics courses are generally considered by students as being challenging, investigating students' learning motivation or attitude is hence an important research focus. It is also reasonable that "skill" was seldom discussed, since it is less relevant to the objectives of mathematics education.

6. Conclusions

In sum, it was found that the advancement of AI and computer technologies has encouraged researchers to conduct diverse AIME studies [7,39–41,59]. Based on the findings and the above discussion, some suggestions for AIME research are given as follows:

- It is suggested that researchers consider using AI applications to provide students with personalized guidance or support, and to investigate the impacts of AI-based learning approaches in mathematics education research.
- It would be innovative to use EDM to investigate the factors affecting students' learning outcomes and to find associations between students' learning behaviors and performances.
- It could be valuable to adopt relevant AI applications in learning activities of advanced mathematics programs, such as geometry and topology, applied mathematics, mathematics literacy and cross-disciplinary (e.g., STEM) courses.
- It is important to consider how AI applications benefit those seldom-adopted sample groups in mathematics education, such as teachers and senior high school students.
- In addition to quantitative analysis, it is important to encourage researchers to conduct qualitative methods to collect learners' feedback on AI-supported mathematics learning and to analyze learners' perceptions in depth.
- It would be valuable to develop adaptive mathematics learning environments via the collaboration of mathematics education, educational technologies and computer science researchers.
- It could be interesting to employ modern AI technologies, such as deep learning, in mathematics education. Although the related AI applications, such as image recognition and voice recognition, might be directly relevant to mathematics content, they can benefit learners from other perspectives, such as providing visually impaired students with a supportive interface.
- It is important to investigate the effectiveness of using AI in mathematics learning activities from different perspectives by taking rarely considered research foci into account, such as cognitive load, collaboration and communication competences and learning anxiety.

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